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FINAL REPORT

RELIABILITY OVERHAUL MODEL
INTERIM REPORT

WALTER A. RUGG

AUGUST 1989

Distribution Unlimited

U.S. ARMY ARMAMENT,
MUNITIONS AND CHEMICAL COMMAND
SYSTEMS ANALYSIS OFFICE
ROCK ISLAND, ILLINOIS 61299-6000

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This report documents the development of a proof of principle model which assists in the development of Depot Maintenance Work Requirements by determining the reliability of equipment. The report includes a discussion of the algorithms as well as the data structure used by the model. Potential uses of the model and its performance are also discussed.

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RELIABILITY OVERHAUL MODEL REPORT

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Systems Analysis Office
Rock Island, IL 61299-6000

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SUMMARY

This report discusses the initial modeling effort to develop a tool which can assist in the development of depot maintenance work requirements by determining the inherent reliability of equipment. A Monte Carlo simulation was developed. The input for the simulation includes a block diagram, a time to failure distribution or failure and suspension data for each block in the diagram, and the age of each block. The primary output of the simulation is a numerical time to failure distribution for the system. This distribution is used to generate a graph of the time to failure distribution, a histogram of the ages at which the system fails, a graph of hazard rates, a total time on test plot and an graph of the expected cost of various replacement policies if cost data is available.

The algorithms and data structure used in the model are discussed as well as the potential uses of the model and its performance.

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FIGURES

1. Introduction

a. Background

This effort was initiated as the result of a proposal made by Gerald Moeller, while a LOGAMP trainee on assignment to the Maintenance Directorate at the Headquarters U.S. Armament, Munitions and Chemical Command. The original proposal was to construct a model, which would support overhaul decisions at depot shops. After a series of discussions between the Maintenance Directorate and the Systems Analysis Office, it was decided that the model should be pursued as an aid to depot maintenance work requirement development, rather than a tool for the depot shop foreman.

b. Objective

The initial objective was to develop a proof of principal model capable of generating hazard rates and assessing a system's reliability once repairs were completed. This model would provide a tool to assist in the identification of components, which should be rebuilt, replaced or left alone, when the system is overhauled. The model had to be structured in a flexible manner so it could readily host the modeling of any weapon system in a manner which enables combining the reliabilities of the various components or sub-assembilies in the peculiar way which reflects that end-item's true reliability. The model also had to be constructed in a manner which would allow personnel having limited computer experience to use it productively. The objective of this report is to summarize the modeling efforts to date.

2. Methodology

a. Selection of Computer Language and Method

The model was coded in Turbo Pascal because it supports recursion, dynamic memory allocation and the graphic routines required by the algorithms used in the model.

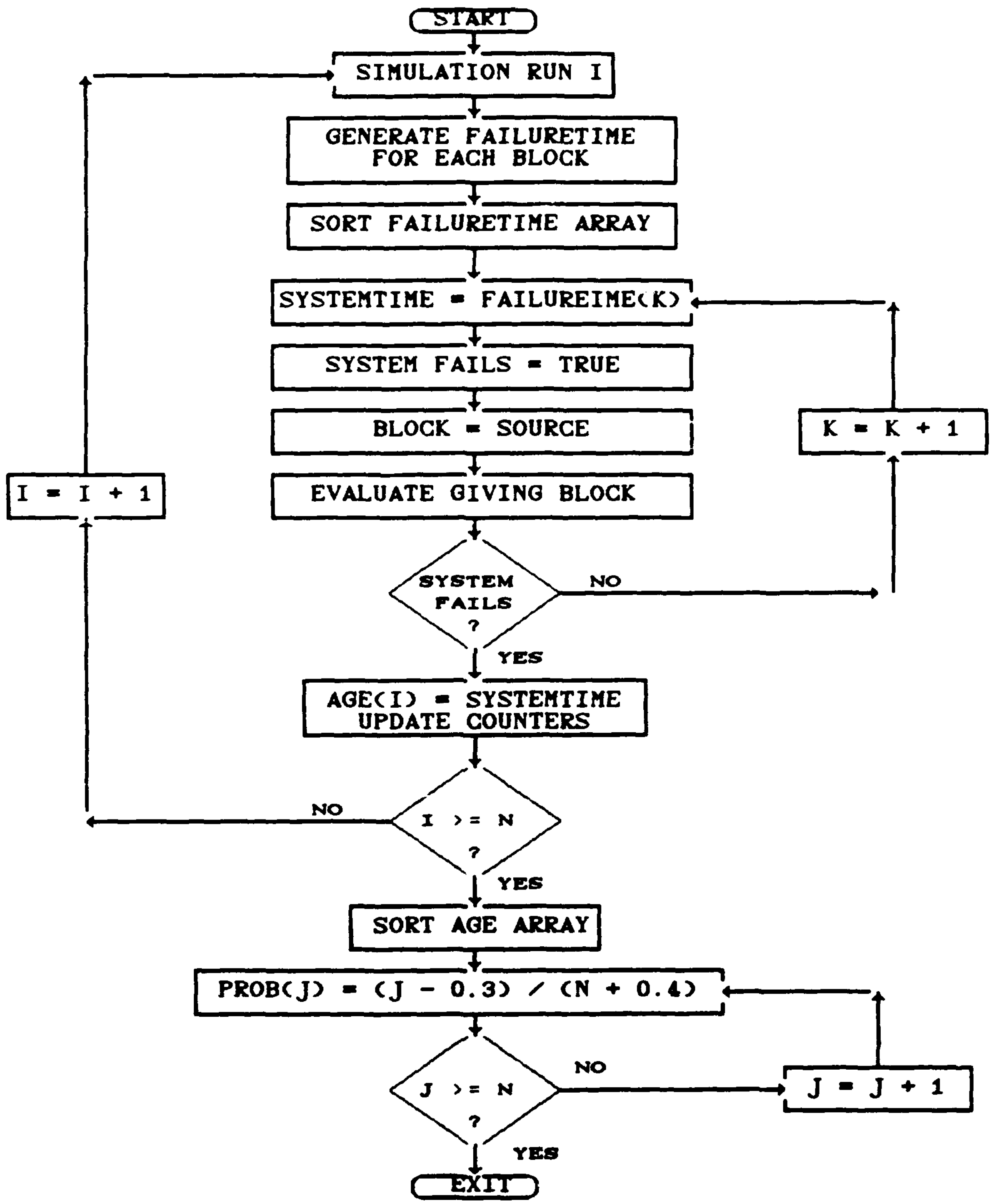
In order to achieve its objectives, the model would have to support the parsing and analysis of block diagrams to estimate system reliability.

Two modeling methods were considered, Monte Carlo simulation and conventional probability analysis. Monte Carlo simulation was selected because of its flexibility and the difficulty in automating the process of converting a block diagram into equations for models more complex than a series/parallel system. The simulation (like all models) is a blend of algorithms and data structures, which are explained in the main body of this report. In addition to the simulation, the program also contains a unit to analyze raw data and a unit to produce graphic output. The algorithms used in these units and the algorithms used to generate the random variables used in the simulation are given in the appendixes.

b. Simulation Algorithm

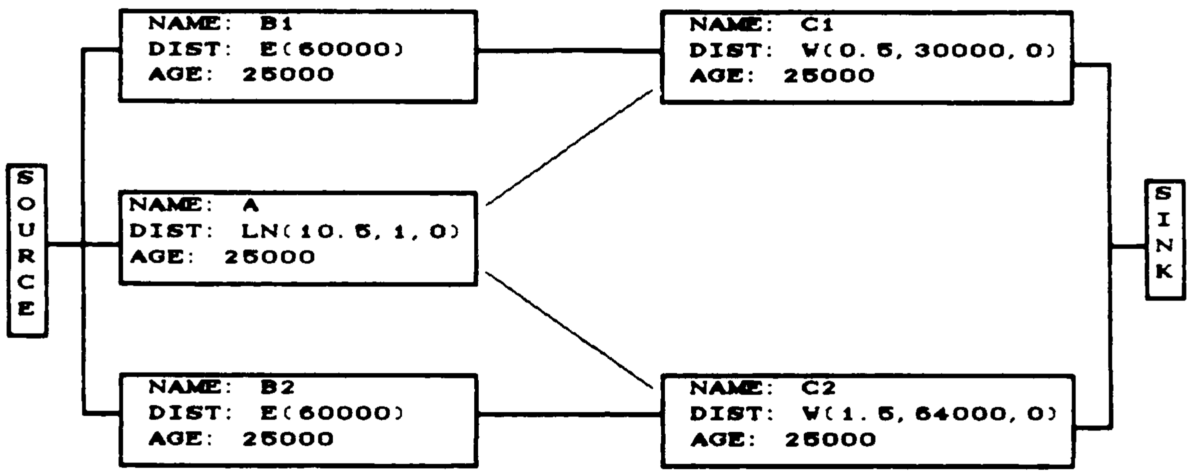
The Monte Carlo method involves random sampling from the conditional time to failure distributions of the components making up the block diagram and using these times to determine the time at which the system fails. The procedure is repeated many times until enough failure times are obtained to estimate the system reliability. The resulting data can be used to

FIGURE 1. FLOWCHART FOR SIMULATION



estimate parameters for candidate time to failure distributions for the system. A flowchart of the logic used in the simulation is provided in Figure 1. In addition, the procedure is illustrated by the following example. The mission reliability diagram is given in Figure 2. The mission requirement would be that equipment A and either equipment C1 or C2 work, or that equipment B1 and C1 work, or that B2 and C2 work for success. The time to failure distribution and age of each component is

FIGURE 2. MISSION BLOCK DIAGRAM



provided in each block. The age of components can be measured in any appropriate unit, such as hours or miles, but the units must be the same for each block. Source and sink blocks have been added to the diagram because they are required by the model's data structure. The source block starts the simulation and the sink block ends the simulation.

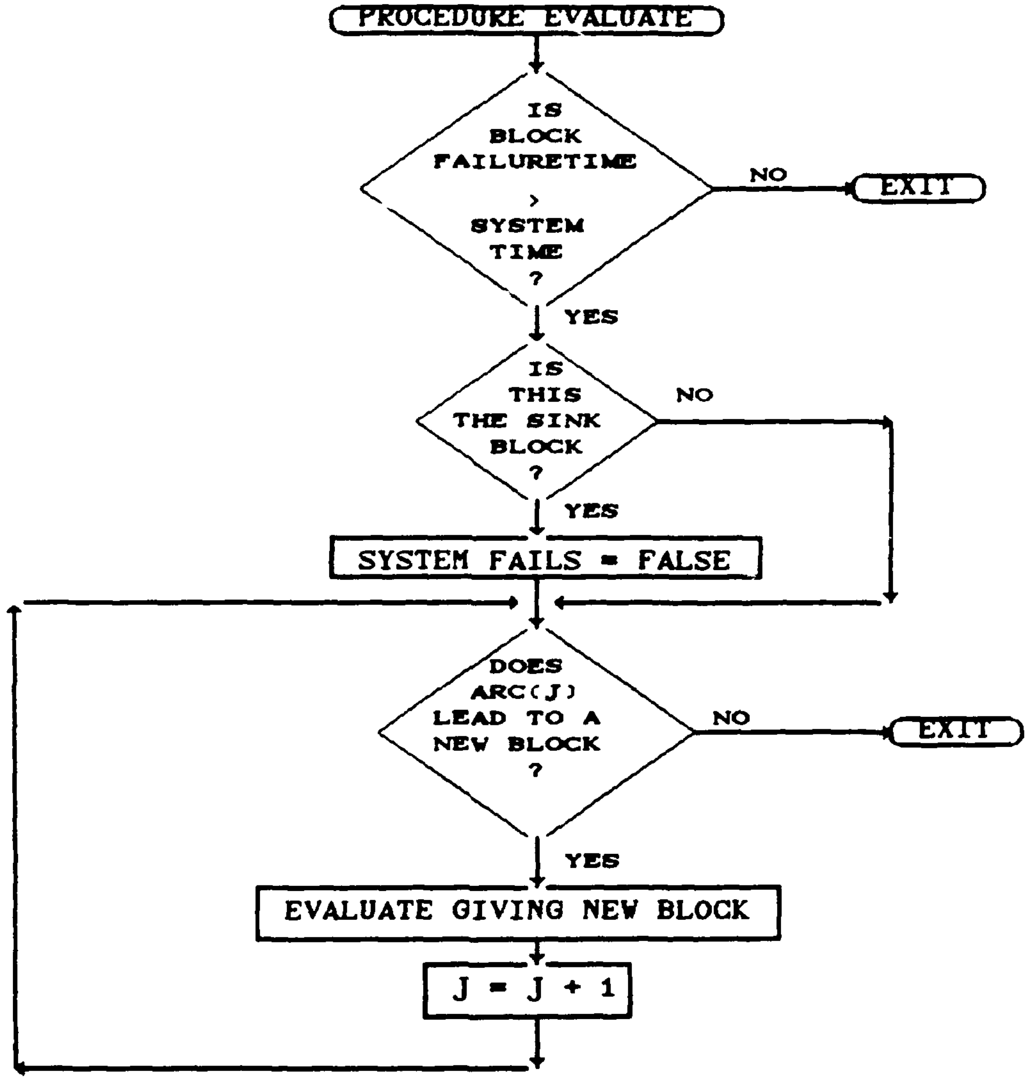
The model first generates random numbers between 0.0 and 1.0 and uses them , with the algorithms in Appendix B,. to calculate a time to failure for each component. These times are given in Table 1.

TABLE 1. TIME TO FIRST FAILURE FOR COMPONENTS.

BLOCK	TIME
A	3162.36
B1	46487.67
B2	10291.19
C1	105741.06
C2	97571.61

The component failure times are sorted and the system time set equal to the minimum component failure time. Using the

FIGURE 3. FLOWCHART FOR PROCEDURE EVALUATE GIVEN BLOCK



procedure Evaluate (see Figure 3.), the model determines if a success path from source to sink can be found among the failed and nonfailed equipments. The model first assigns a value of true to the boolean variable SYSTEMFAILS. It then evaluates the blocks by comparing their time to fail with the system time. If the time to fail is greater then the system time, the procedure also evaluates the blocks for which the current block is the origin node. If the block being evaluated is the sink block and it's time to failure is greater than the system time, the variable SYSTEMFAILS is set to false. The system time is updated to the time of the next component failure and the process continued until a success path does not exist. Table 2 shows the various system times and the condition of the system at each time.

TABLE 2. SUCCESS/FAILURE ARRAY FOR BLOCK DIAGRAM

SYSTEM TIME	A	B1	B2	C1	C2	SYSTEM
3162.36	F	S	S	S	S	S
10291.19	F	S	F	S	S	S
46487.67 *	F	F	F	S	S	F
97571.16	F	F	F	S	F	F
105741.06	F	F	F	F	F	F

* The system fails at this point. No futher evaluation would take place during this iteration. S ≡ SUCCESS F ≡ FAILS

When the system fails, the model records the age of the system at failure, updates counters and repeats the process until the number of iterations performed equals the number required. Once the required iterations are completed, the ages are sorted and the associated probabilities estimated, using median ranks. The two keys to performing this simulation are the recursive procedure Evaluate and the data structure, which allows the model to traverse the block diagram.

c. Data Structure

The pri[illegible] building blocks of the simulation's data structure are record[illegible]sociated with each component. These records are a combination of simple-type data into a new data type, which has two advantages. All data elements for a single record are logically connected to each other. Also, some operations, such as assignment, can be performed on the entire record, eliminating the need to refer to each element of the record.

These records are made up of two kinds of data elements, those that contain information about a system component and those that point to other records. The pointers connect blocks to form a linked list for data editing and a tree, which is used to traverse the block diagram. Appendix D contains the data elements which make up each record and a brief discussion of how they are used.

3. Data Requirements

The data requirements for this or any model meeting the above objectives include the following.

a. A block diagram for the equipment to be overhauled.

b. A time to failure distribution for each block or failure and suspension data which can be used to generate the failure distributions. The failure distributions currently available are exponential, uniform, weibull, normal, lognormal and a constant probability of failure.

c. The age of each block.

d. The cost of scheduled replacement and of field failures if an expected cost distribution is required.

Availability of this data is the major obstacle to the implementation of this type of model.

4. Model Performance

a. Accuracy

All Monte Carlo estimates have an associated error band. The larger the number of iterations the more precise the estimate is. A 100(1.0 - α) percent confidence interval (CI) on a given reliability estimate can be constructed, using the following formula, which comes from the normal approximation of the binomial probability density function.

$$\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\,(1 - \hat{p})}{N}}$$

where $Z_{\alpha/2}$ is the 100(1 - α/2) percentile point of the standard normal distribution, $\hat{p}$ is a point estimate of the reliability and N is the number of iterations.

The model's output for the bridge diagram in Figure 1. was compared with the actual time to failure distribution and the maximum absolute error (MAE) recorded. The results are given in Table 3 with the largest error expected for 90 and 95 percent confidence intervals.

Table 3. MODEL OUTPUT VS ACTUAL RELIABILITIES

		MAXIMUM WIDTH	
# Iterations	MAE	90% CI	95% CI
1000	0.02476	0.03010	0.03542
2000	0.01505	0.02191	0.02504
3000	0.01472	0.01789	0.02045
4000	0.01356	0.01550	0.01771
5000	0.01625	0.01386	0.01584
6000	0.01222	0.01265	0.01446
7000	0.01144	0.01171	0.01339

A confidence interval on the error in the estimate of the mean time to failure can be obtained using $E = \pm Z_{\alpha/2} \frac{\hat{\sigma}}{N^{1/2}}$, where $\hat{\sigma}$ is an estimate of the standard deviation.

Since this is a proof of principal model, the number of iterations has been preset to 1000. If this model becomes a standing model it should be modified to allow the analyst to set the number of iterations as well as the random number seeds and other parameters making up the simulation environment. Also variance reduction techniques, such as stratified sampling, should be employed.

b. Execution Time

The length of time it takes to run the model depends on the number of blocks in the diagram, the distributions used and the complexity if the block diagram. The run times for various configurations is given in Table 4.

Table 4 EXECUTION TIME

Configuration	Time
1. One Block with various Distributions	4-6 sec
2. The Bridge diagram in Figure 2.	22 sec
3. Five Exponential Distributions in parallel	19 sec
4. Five Exponential Distributions in series	14 sec
5. 100 blocks in series with Uniform distributions. The last blocks had a maximum time to failure less than the mimimum of any other block.	3 min 53 sec
6. The same configuratioin as number 5, but with 1000 blocks.	42 min 14 sec

5. Potential Uses of the Model

a. Support of Overhaul Decision Making

The current model meets the initial objectives of generating hazard rates and assessing a system's reliability once repairs are completed. It can be used to support decision making concerning when (or if) overhaul should take place, by generating time to failure distributions for various levels of overhaul. The model can support decisions based on mission reliability, change in the MTTF or on the basis of cost. However, the current configuration of the model requires several runs, with the analyst manually modifying the data. A possible enhancement to the model would be the generation of a list of components ordered by the effect of their overhaul on the the system reliability.

b. General Tool for Reliability Analysis.

Because the information generated by the model is fundamental to any analysis of the reliability of a system or component, the model can be used to support reliability analysis of block diagrams or failure and suspension data. The model can generate the following information for a block diagram.

(1) A numerical time to failure distribution
(2) Numerical conditional time to failure distribution
(3) Numerical reliability distribution
(4) Hazard rates
(5) Mean time to failure
(6) The probability each component outlives the system

The model can support the following data analysis.

(1) Estimation of parameters for time to failure distributions.
(2) Nonparametric estimation of hazard rates.
(3) Calculation of mortality curves using ranking.

c. A Base for Further Modeling

The current model provides a foundation for models intermixing maintenance policies and the inherent reliability of hardware. Such models could be constructed in two ways. The preferable method would be to use the data generated by the model to construct a time to failure function. This could be done using the current raw data analysis unit of the model or by fitting various functions to portions of the data and generating a function in a piecewise manner. Once a distribution was constructed, it could be used (with other information) to generate such things as the effect of corrective and preventive maintenance on the reliability of a system and estimates of the steady state availability of maintained systems.

The second method would be to continue to build on the existing simulation. Maintenance actions would be scheduled, in the simulation, in a fashion similar to the way component failures are scheduled in the current simulation. The effect of maintenance actions would be modeled by generating new failure times for the components on which maintenance was performed and recording such things as down time to allow the model to estimate availability as well as the effect of the maintenance on reliability. Such a model would allow analysis of systems too complex to work with analytically.

6. Conclusions

The development of the proof of principal model, demonstrates the analytical feasibility of computer models performing reliability analysis, which can support decision making regarding the type of maintenance that should be performed. However, such analysis should only be carried out in conjunction with an engineer familiar with the equipment being studied. An analysis without this expertise, could easily result in incorrect conculusions and poor maintenance planning.

The question of the availability of accurate data to run such models seems to be the major objection to their use. The lack of good data can be a problem, especially early in the life of a weapon system. However, plans are being made and must be made with the data available. The important question is whether the model will or will not make better use of the available data.

We believe it will. The primary product of the model, a numerical time to failure function, is fundamental to the analysis of the reliability and maintainability of any system. In all but the most simple cases reliability analysis entails rather complicated mathematical formulations and tedious computations. This type of model provides a structured method for carrying out such analysis with a minimum of simlifying assumptions, while providing a tool for the systematic analysis of failure data and updating assessments of the reliability of components as well as the system.

APPENDIX A MORTALITY CURVE CONSTRUCTION

A mortailty curve gives values for the proportion of a population that fail (P_i) before a given time. The model calculates the points on a mortality curve using ranking. It requires a data set consisting of the age at which failure or suspension takes place (X_i), the number of items exposed in the interval $X_i - X_{i-1}$ (E_i), and the number of items which failed in the interval, $X_i - X_{i-1}$, (F_i) for $i = 1, 2, 3, \ldots, N$. The following algorithm is used.

a. Set $R = 1.0$ and $MAXE = Max(E_1, E_2, \ldots, E_N)$. Order the data chronologically.

b. Set $R = R(1 - \frac{F_i}{E_i})$ and $P_i = 1 - R$, for $i = 1, 2, \ldots, N$.

c. Set $P_i = \frac{MAXE\ P_i - 0.3}{MAXE + 0.4}$, for $i = 1, 2, \ldots, N$.

For uncensored data, this algorithm reduces to $P_i = \frac{i - 0.3}{N + 0.4}$, which is a commonly used appromiximation of the median rank of the i^{th} failure.

APPENDIX B DISTRIBUTIONS SUPPORTED BY THE MODEL

1. This appendix contains information concerning the distributions supported by the model. For each distribution, the following information is provided.

a. A brief discription of how the distribution is usually used in reliablilty modeling.

b. The conditional time to failure distribution.

c. The algorithm used to generated random variables from the conditional time to failure distribution.

d. The algorithm used to estimate the distribution's parameters given a set of failure and suspension data.

2. Exponential $F(x) = 1 - e^{-x\lambda}$

a. The exponential distribution is the most widely used in reliability . It can be used to model random component failures and has a constant failure fate of λ.

b. The probability of failure at time (x+s) given the component has survived to time x is given below.

$$P(X > x+s|X > x) = P(X > x + s) / P(X > x)$$
$$= \frac{e^{-\lambda(x + s)}}{e^{-\lambda x}}$$

$$P(X > x+s|X > x) = e^{-\lambda s}$$

$$P(X \le x+s|X > x) = 1 - e^{-\lambda s}$$

c. Random variables for the conditional exponential distribution are generated using the inverse transform method.

(1) Generate $U \sim U(0,1)$

(2) Set $s = \lambda \ln(U)$

d An estimate of λ is made by applying ordinary least squares (OLS) to the linerized distribution and then using the derivative of the sum of squared errors (SSE) to search in the neighbourhood of $\hat{\lambda}$ for an estimate of λ which minimizes the SSE.

(1) Set $Y = -\ln(1.0 - P(X \le x_i))$; and $x_i' = \ln(X_i)$ for $i = 1, 2, \ldots N$ Where N equals the number of observations. Generate $\hat{\lambda}$ for $Y = \lambda X'$ using OLS. Set $L = 0$ and $U = 2\hat{\lambda}$.

(2) Set $\frac{d\ SSE}{d\ \hat{\lambda}} = \sum_{i=1}^{N} -2\ X_i e^{-\hat{\lambda}X_i} \left\{P(X \le x_i) - 1 + e^{-\hat{\lambda}X_i}\right\}$

(3) If $\frac{d\ SSE}{d\ \lambda} \ge 0.0$ then set $U = \hat{\lambda}$ Otherwise, set $L = \hat{\lambda}$.

Set $\hat{\lambda} = L + (U - L) / 2.0$.

(4) If $\frac{U - L}{\lambda} \leq 0.00001$ stop. Otherwise, go to 2.

3. Uniform $F(X) = \dfrac{(X - a)}{(b - a)}$

a. This distrubition is not used very often in reliablility. It could be used in cases where the only thing known about a component is its maximum life.

b. $P(X > x+s | X > x) = P(X > x + s) / P(X > x)$

$$= \frac{1 - \frac{x + s - a}{b - a}}{1 - \frac{x - a}{b - a}}$$

$$= \frac{b - x - s}{b - x}$$

$$P(X > x+s | X > x) = 1.0 - \frac{s}{b - x}$$

$$P(X \leq X+s | X > x) = \frac{s}{b - x}$$

c. Random variables from the conditional uniform distribution are generated using the inverse transform method.

(1) Generate $U \sim U(0,1)$

(2) set $s = U (b - x)$

d. Estimates of the parameters of the Uniform distribution are made by applying OLS to failure and suspension data.

(1) Set $Y_i = P(X \leq x_i)$.

(2) Estimate a and b for $Y_i = \frac{1}{(b - a)} x_i - \frac{a}{(b - a)}$ using OLS.

4. Weibull $F(X) = 1 - e^{-[(X-\gamma)/\eta]^{\beta}}$

a. The weibull distribution is the most general one used to model reliability. It can be used to model initial failures or wearout failures. Its failure rate depends on the shape parameter, β.

(1) If $\beta > 1$ increasing failure rate.

(2) If $\beta < 1$ decreasing failure rate.

(3) If $\beta = 1$ constant failure rate.

b. $P(X > x+s | X > x) = P(X > x+s) / P(X > x)$

$$P(X > x+s | X > x) = \frac{e^{-[(x+s-\gamma)/\eta]^{\beta}}}{e^{-[(x-\gamma)/\eta]^{\beta}}}$$

$$P(X \le x+s \mid X > x) = 1 - e^{-[(x+s-\gamma)/\eta]^{\beta} + [(x-\gamma)/\eta]^{\beta}}$$

c. Random variables from the conditional weibull distribution are generated using the inverse transform method.

(1) Generate $U \sim U(0,1)$

(2) Set $s = \eta[-\ln(1-U) + ((x-\gamma)/\eta)^{\beta}]^{1/\beta} + \gamma - x$

d. Estimates of γ, η and β are made by combining a golden section search and OLS to obtain estimates which minimize the SSE.

(1) Set $B = 0.0$. Set $U = \min(X_1, X_2, \ldots X_N)$. If $\min(X_1, X_2, \ldots X_N) > 0$ then set $T = \min(X_1, X_2, \ldots X_N)$. Otherwise set $T = 1.0$. Set $\gamma_1 = 0.382\ (U - B) + B$.

Set $X_i' = \ln(X_i - \gamma_1)$ and $Y_i = \ln\left\{\ln\left\{\frac{1.0}{1 - P(X \le X_i)}\right\}\right\}$ for $i = 1, 2, \ldots, N$. Generate OLS estimates for $Y_i = \beta X_i' + \beta \ln(\eta)$ and SSE_1.

(2) Set $\gamma_2 = 0.618\ (U - B) + B$. Set $X_i' = \ln(X_i - \gamma_2)$ and $Y_i = \ln\left\{\ln\left\{\frac{1.0}{1 - P(X \le X_i)}\right\}\right\}$ for $i = 1, 2, \ldots, N$. Generate SSE_2 and OLS estimates for $Y_i = \beta X_i' + \beta \ln(\eta)$.

(3) If $SSE_1 > SSE_2$

then set $B = \gamma_1$; $\gamma_1 = \gamma_2$; $\gamma_2 = 0.618\ (U-B) + B$; $X_i' = (X_i - \gamma_2)$ and $SSE_1 = SSE_2$. Generate OLS estimates and SSE_2

Otherwise, set $U = \gamma_2$; $\gamma_2 = \gamma_1$; $\gamma_1 = 0.383\ (U - B) + B$; $X_i' = (X_i - \gamma_1)$ and $SSE_2 = SSE_1$. Generate OLS estimates and SSE_1.

(4) If $\left|\frac{(\gamma_2 - \gamma_1)}{T}\right| > 0.0001$ go to 3. Otherwise, stop.

5. Normal $F(X) = \int_{-\infty}^{X} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}\,dx$

a. No closed form exists for this distribution. Its failure rate is a monotonically increasing function of X. The normal distribution can be used to model weareout and stress failure where the random variable is stress rather than time.

b. There is no closed form of the conditional time to fail distribtion.

c. Random variables from the conditional normal distribution, $P(X \le x+s \mid X > x)$, are generated using a standard normal transformation and the inverse transform method.

APPENDIX B DISTRIBUTIONS SUPPORTED BY THE MODEL

(1) Generate $Y = P(X \leq x)$ from the standard normal distribution.

(2) Generate $U' \sim U(0,1)$ and set $U = U'(1 - Y) + Y$

(3) Generate Z associated with U from the standard normal distribution.

(4) set $s = \mu + \sigma Z - x$

d. Estimates for the parameters of the Normal distribution are made by applying OLS to the standard normal distribution associated with failure and suspension data.

(1) Generate Y_i as the inverse standard normal value associated with $P(X \leq X_i)$ for $i = 1, 2, \ldots N$.

(2) Use OLS to estimate parameters for $Y_i = \frac{1}{\sigma} X_i - \frac{\mu}{\sigma}$ and use these parameters to estimate μ and σ.

6. Lognormal $$\int_{-\infty}^{x} \frac{1}{x \sigma \sqrt{2\pi}} \exp \frac{-(\ln x - \mu)^2}{2\sigma^2} dx$$

a. The failure rate of the lognormal distribution initially increases over time and then decreases, approaching zero. It is useful for modeling situations in which early failures dominate.

b.. There is no closed form of the conditional time to fail distribution.

c. Random variables from the conditional lognormal distribution, $P(X > x+s | X > x)$, are generated using an acceptance/rejection method.

(1) Generate $Y \sim N(\mu, \sigma | X > x)$

(2) Set $s = e^{Y+x} - x$

d. Estimate for the paramters of the Lognormal distribution are made by estimating the parameters of the Normal distribution associated with the Lognormal.

(1) Set $X_i' = \ln(X_i)$ for $i = 1, 2, \ldots N$.

(2) Generate Y_i as the inverse standard normal value associated with $P(X \leq X_i')$ for $i = 1, 2, \ldots N$.

(3) Use OLS to estimate parameters for $Y_i = \frac{1}{\sigma} X_i - \frac{\mu}{\sigma}$ and use these parameters to estimate μ and σ.

7. In addition to the above distribution the model also supports use of components with constant (not age dependent) probabilities of failure. Random variables are generated for the components

using the inverse transform method.

a. Generate $U \sim U(0,1)$

b. If $U >$ P(component fails) then set $s = 10^{300}$. Otherwise, set $s = 0.0$.

APPENDIX C GRAPHICS GENERATED BY THE MODEL

FIGURE 4. TIME TO FAILURE DISTRIBUTION

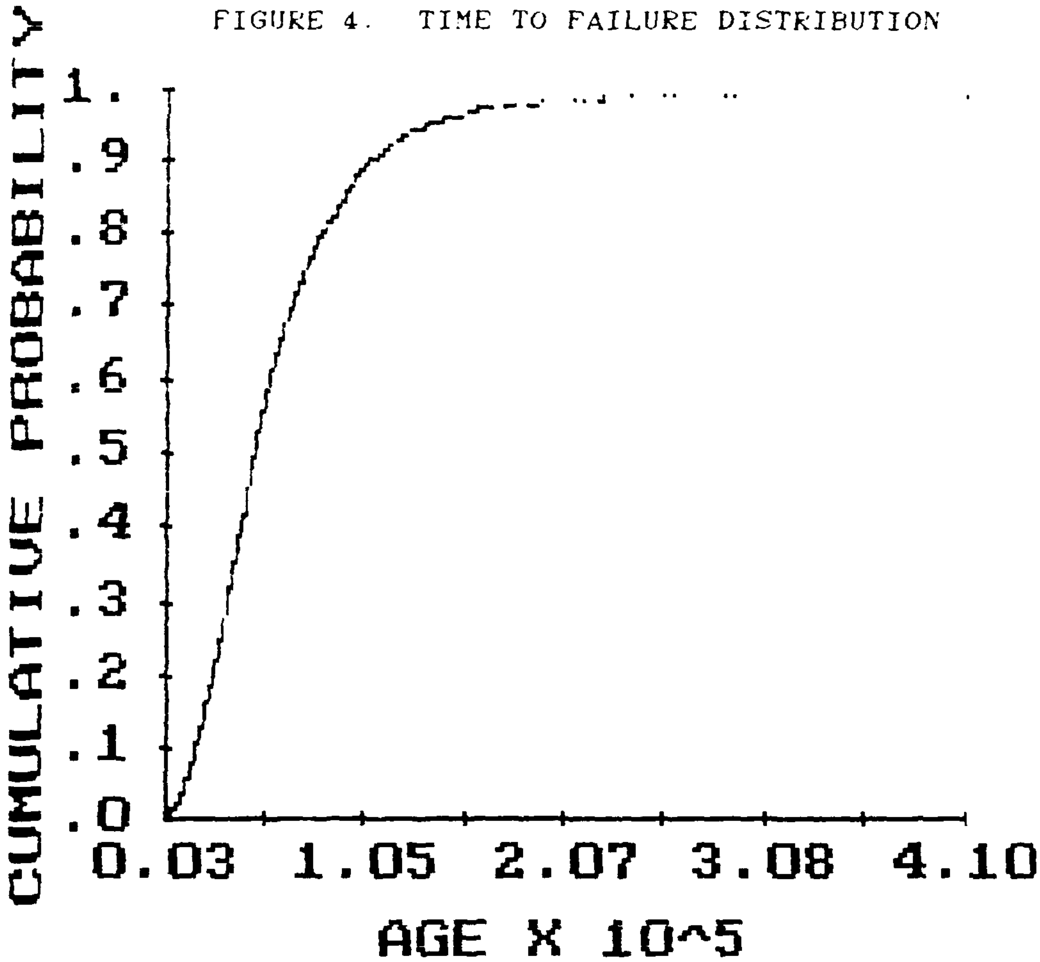


FIGURE 5. HISTOGRAM OF FAILURE TIME FOR BRIDGE

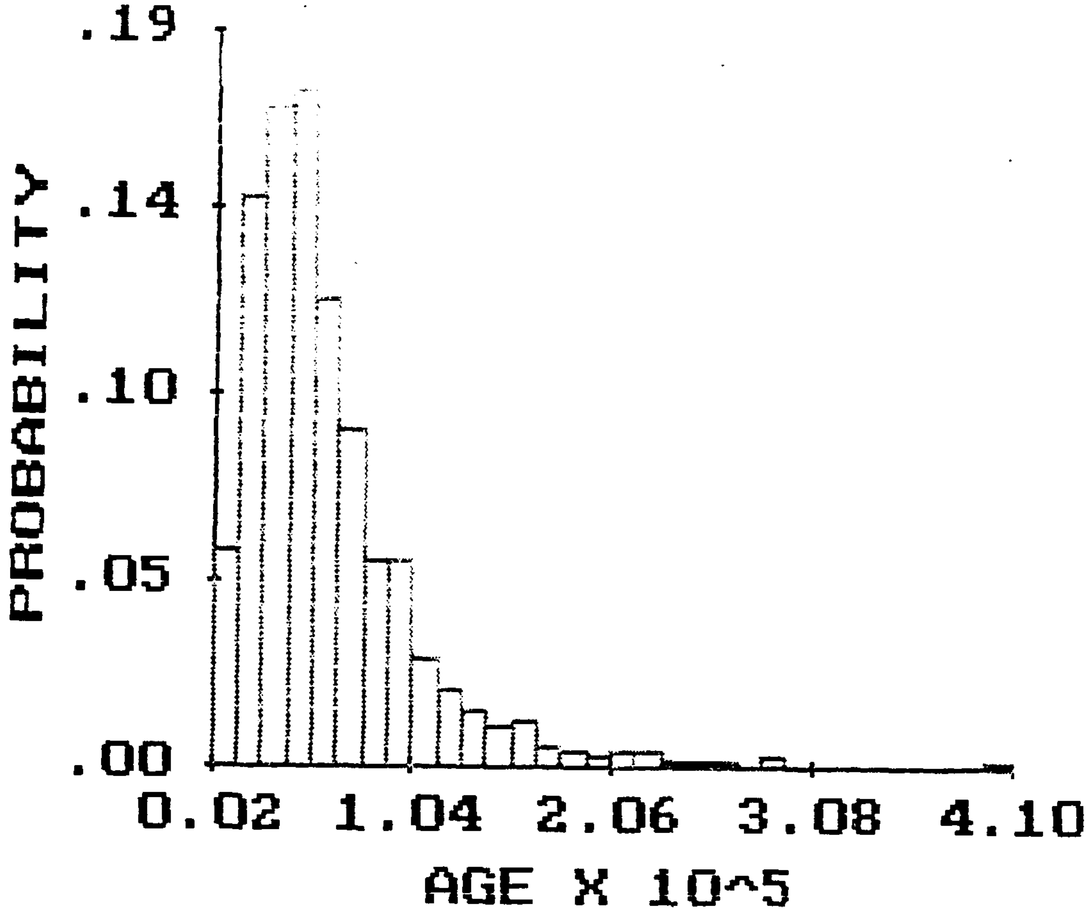
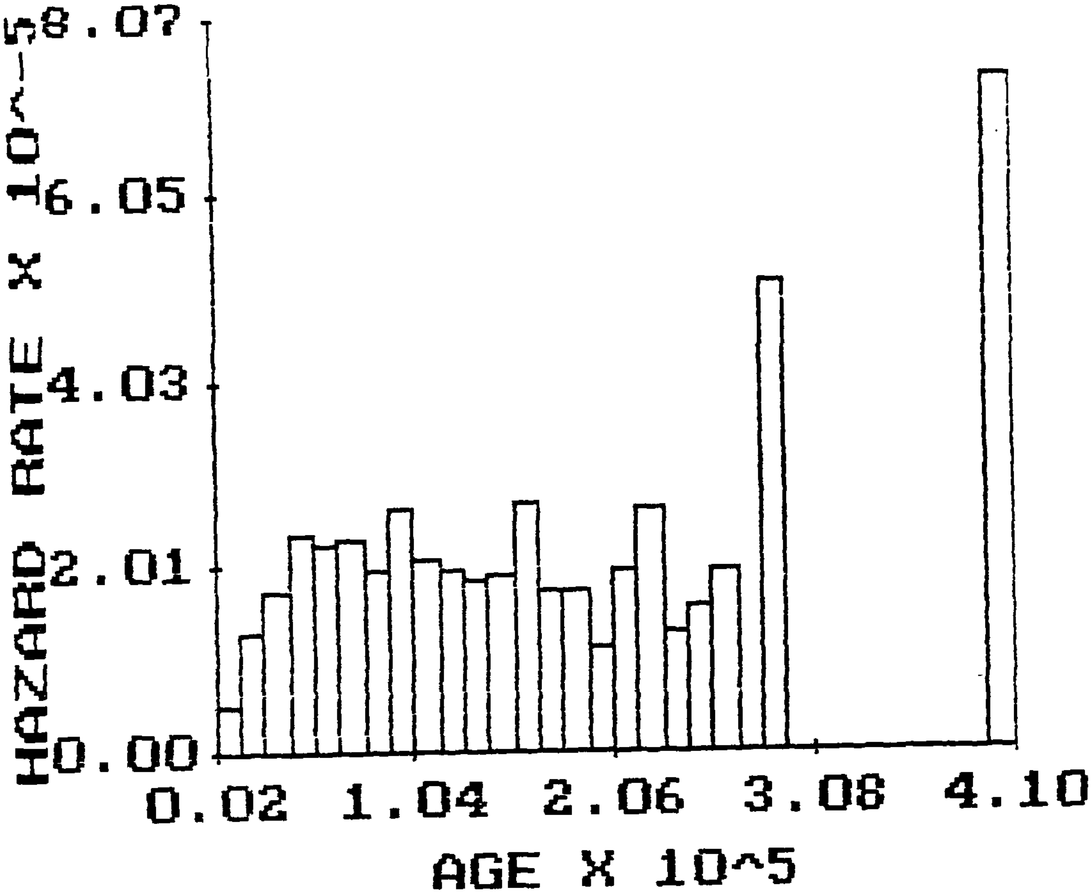
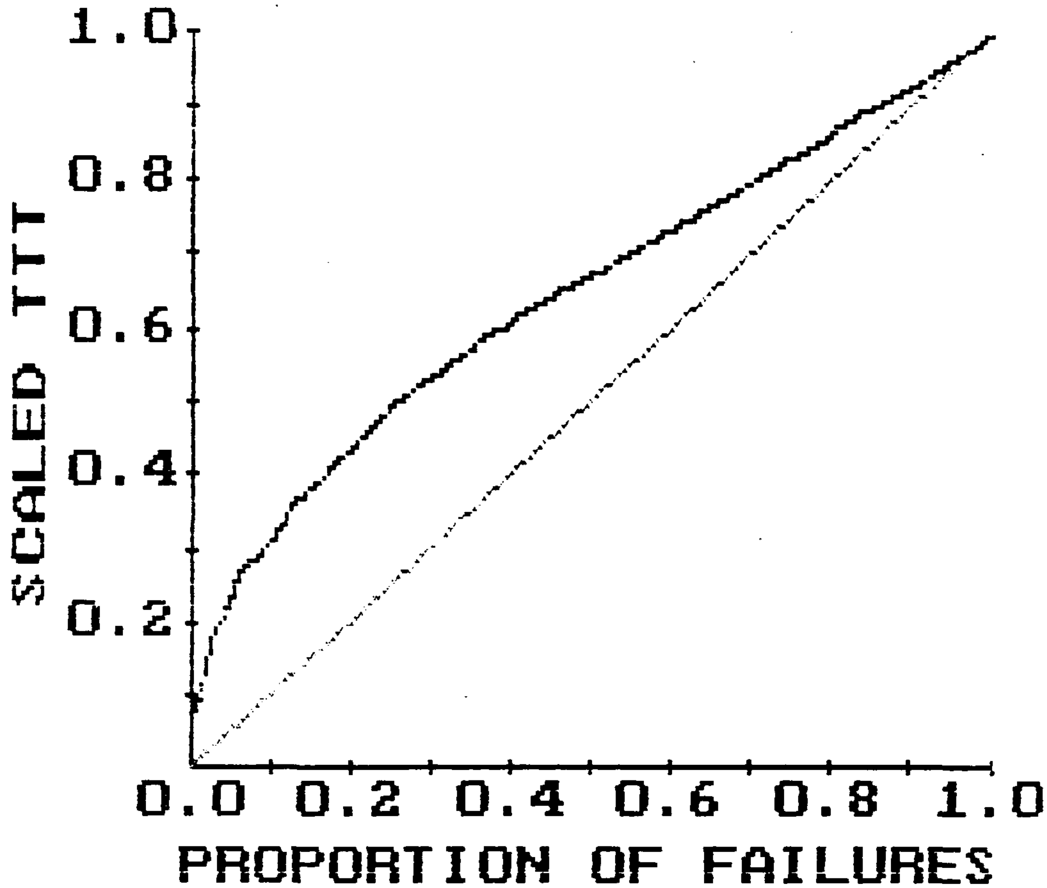


FIGURE 6. HAZARD RATES



$$\text{HAZARD RATE} = \frac{\#\ \text{FAILURES}\ (t, t + \Delta t)}{\#\ \text{SURVIORS AT}\ t + \Delta t}$$

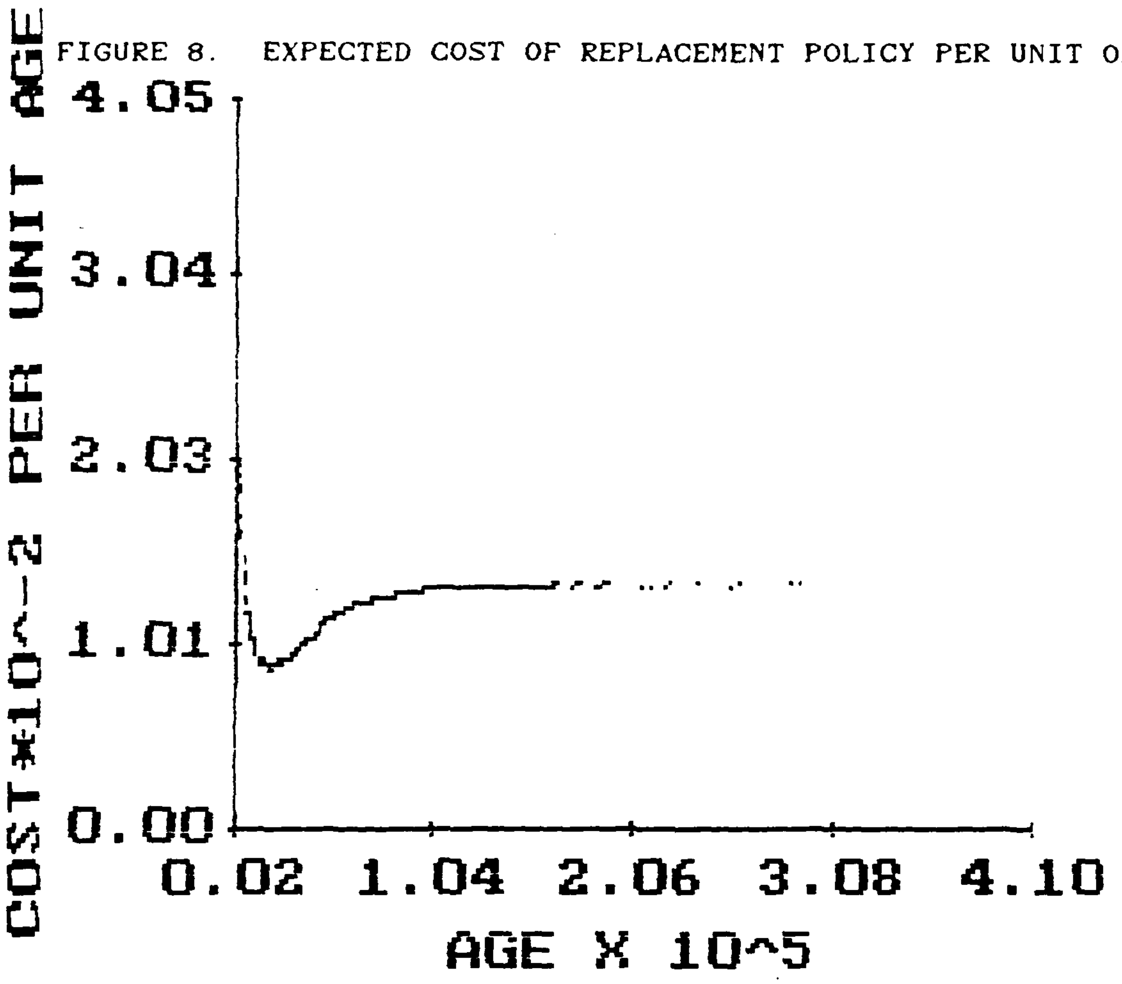
FIGURE 7. TOTAL TIME ON TEST PLOT



$$TTT_I = TTT_{I-1} + \frac{(N - I + 1)\,\Delta T_I}{SCALE}$$

WHERE, $SCALE = \sum_{J=1}^{N} (N - J + 1)\,\Delta T_J$ AND $I = 1, 2, 3, \ldots, N$

FIGURE 8. EXPECTED COST OF REPLACEMENT POLICY PER UNIT OF AGE



$$\text{COST PER UNIT OF AGE} = \frac{E(COST)_I}{E(CYCLE)_I}$$

WHERE, $E(COST)_I$ is the expected cost per cycle for replacement at AGE_I and $E(CYCLE)_I$ is the expected cycle length for replacement at AGE_I.

DATA RECORD FOR BLOCKS

TABLE 5. DATA ELEMENTS FOR BLOCK RECORDS

ELEMENT	TYPE OF DATA ELEMENT		
NAME	STRING OF 10 CHARACTERS		
AGE	REAL NUMBER		
FAILURETIME	REAL NUMBER		
SURVIES	REAL NUMBER		
NEXT	POINTER TO A BLOCK RECORD		
ARC	ARRAY OF 10 POINTERS TO BLOCK RECORDS		
TYPEOFDIST	USER DEFINED DATA TYPE (EXPONENTIAL, FIXED, UNIFORM, WEIBULL, NORMAL, LOGNORMAL)		
DISTRIBUTION	VARIANT RECORD DEPENDENT ON VALUE OF TYPEOFDIS		
	EXPONENTIAL ▶	MEAN TIME TO FAILURE	: REAL
	FIXED ▶	CONSTANT	: REAL
	UNIFORM ▶	MIN, MAX	: REAL
	WEIBULL ▶	SHAPE, SCALE, LOCATION	: REAL
	NORMAL ▶	MEAN, STANDARD DEVIATION	: REAL
	LOGNORMAL ▶	μ, σ	: REAL

Table 5. contains the data elements, which make up each record. The element, NEXT, combines with a pointer variable, FIRSTRECORD, to form the singly linked list which allows sequential access to the block records. The program constructs this list as the information elements of the records are input and insures that each record has a unique element, NAME. The tree, which represents the blocks diagram, is then constructed by inputing the element, NAME, for the records making up the origin and destination nodes of each arc. The program searches the linked list to find the record associated with the element, NAME for the origin node and then searches for the record associated with each destination node and assigns a value to the array, ARC, which points to the destination record.

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